

Summary – Applications of Linear Systems

Representing words with mathematics, then setting up and solving a linear system that models a real-world problem.

Mathematical equations are used to model or *represent* situations in the real world.

If we can model a situation that happens in the real world with one or more equations, and then solve the equations, well, then we have done something truly useful with mathematics.

Example 1

In mathematics, how do we represent the following words?

sum, plus, total + difference, subtract – times ×

Example 2

Write the words below as mathematical expressions.

1. Let x represent the larger of two numbers, and y the smaller. Write expressions for:

- | | |
|--|-----------|
| a) the sum of the two numbers | $x + y$ |
| b) the smaller subtracted from the larger | $x - y$ |
| c) the sum of six times the larger and two times the smaller | $6x + 2y$ |

2. A plane flies at x kilometres per hour in still air. The wind speed is y kilometres per hour. Write expressions for the speed of the plane when it is flying:

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|------------------|---------|
| a) into the wind | $x - y$ |
| b) with the wind | $x + y$ |

3. A fitness centre charges an initiation fee of $\$x$ plus a monthly charge of $\$y$. Write expressions to show how much it costs to be a member for:

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|--------------|-----------|
| a) 7 months | $x + 7y$ |
| b) 15 months | $x + 15y$ |

4. Tickets to a college swim meet cost \$10 for general admission and \$5 for students. There were x general admission tickets and y student tickets sold. Write expressions for:

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|---|------------|
| a) the total number of tickets sold | $x + y$ |
| b) the revenue (money made), in dollars, from general admission tickets | $10x$ |
| c) the revenue, in dollars, from student tickets | $5y$ |
| d) the total revenue from all the tickets | $10x + 5y$ |

Example 2 (continued)

5. Traci invested $\$x$ at 7% and $\$y$ at 6%.

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|--|-----------------|
| a) write 7% as a decimal | 0.07 |
| b) write 6% as a decimal | 0.06 |
| c) write an expression for: | |
| i. the total amount of money Traci invested | $x + y$ |
| ii. the interest Traci earned at 7% in one year | $0.07x$ |
| iii. the interest Traci earned at 6% in one year | $0.06y$ |
| iv. the total interest Traci earned in one year | $0.07x + 0.06y$ |

Solving real-world problems modelled by linear systems

To solve these types of problems:

1. Read the problem carefully, underline what you are being asked to find, and assign variables to these “unknowns” (write “let” statements).
2. Write two equations that describe the relationship between the unknowns.
3. Solve the system of equations.

Example 3

Solve the problem described below.

At a banquet hall to be used for a wedding, there will be two possible dishes served to guests: a chicken dish that costs \$16 and a beef dish that costs \$18. For the 300 guests attending the wedding, the total cost of preparing the food is \$5256. How many chicken dishes will need to be prepared and how many beef dishes will need to be prepared?

Let x be the number of chicken dishes that need to be prepared.

Let y be the number of beef dishes that need to be prepared.

$$x + y = 300 \quad \leftarrow \text{number of dishes}$$

$$16x + 18y = 5256 \quad \leftarrow \text{cost of preparing dishes}$$

$16x$ is the cost of the chicken dishes;

$18y$ is the cost of the beef dishes.

Hint: If you have two unknowns (as we do here, x and y), we also need two equations to be able to solve the system.

Now we have two equations and two unknowns in each; in other words, a system of linear equations. We can solve this by graphing, by substitution, or by elimination. This is left as an exercise for you to complete! When you are done, you should find that 72 chicken dishes need to be prepared, and 228 beef dishes need to be prepared.