

Summary – Solving Linear Systems by Substitution

Solving a linear system algebraically: isolate a variable in one equation, substitute into the other equation, then check the solution in both equations.

The method of substitution

To solve a linear system by **substitution**:

1. In one equation, isolate a variable. Choose a variable with a coefficient of 1 (or -1) if you can.
2. Substitute that expression into the other equation, then solve for the one variable that is left.
3. Substitute the value you found into either original equation to find the other variable.
4. State the solution as an ordered pair (x, y) : the point of intersection.

Example 1

Find the point of intersection (that is, solve for x and y).

A $2x + y = 2$

B $3x - 2y = 10$

From **A**: $y = -2x + 2$

Substitute into **B**:

$$\begin{aligned} 3x - 2y &= 10 \\ 3x - 2(-2x + 2) &= 10 \\ 3x + 4x - 4 &= 10 \\ 7x - 4 + 4 &= 10 + 4 \\ 7x &= 14 \\ \frac{7x}{7} &= \frac{14}{7} \\ x &= 2 \end{aligned}$$

Substitute $x = 2$ into **A**, the other equation:

$$\begin{aligned} 2x + y &= 2 \\ 2(2) + y &= 2 \\ 4 + y - 4 &= 2 - 4 \\ y &= -2 \end{aligned}$$

$\therefore$ the solution, or point of intersection, is $(2, -2)$.

This is only half a solution. We need an ordered pair (x, y) : the intersection point.

We have x , so how do we find y ?

Example 2

Find the point of intersection (that is, solve for x and y).

A $3x + 6y = 4$

B $x - 2y = 1$

From **B** (since it has a term with a coefficient of 1): $x = 1 + 2y$

Substitute into **A**:

Substitute $y = \frac{1}{12}$ into **B**, the other equation:

$$\begin{aligned} 3x + 6y &= 4 \\ 3(1 + 2y) + 6y &= 4 \\ 3 + 6y + 6y &= 4 \\ 3 + 12y - 3 &= 4 - 3 \\ 12y &= 1 \\ \frac{12y}{12} &= \frac{1}{12} \\ y &= \frac{1}{12} \end{aligned}$$

$$\begin{aligned} x - 2y &= 1 \\ x - 2\left(\frac{1}{12}\right) &= 1 \\ x - \frac{2}{12} &= 1 \\ x - \frac{1}{6} + \frac{1}{6} &= 1 + \frac{1}{6} \\ x &= \frac{6}{6} + \frac{1}{6} \\ x &= \frac{7}{6} \end{aligned}$$

Leave your answer as a fraction in lowest terms, unless the decimal equivalent terminates.

Use improper fractions, instead of mixed fractions.

$\therefore$ the solution, or point of intersection, is $\left(\frac{7}{6}, \frac{1}{12}\right)$.

Example 3

Check the solution to the linear system.

By the method of substitution, you found that the solution to the system

A $x - y = 7$

B $2x + y = -10$

was $(-1, -8)$.

Check the solution. You must do a left-side / right-side check. **Form matters!**

Equation A

Equation B

$$\begin{aligned} \text{L.S.} &= x - y & \text{R.S.} &= 7 \\ &= (-1) - (-8) \\ &= -1 + 8 \\ &= 7 \end{aligned}$$

$$\begin{aligned} \text{L.S.} &= 2x + y & \text{R.S.} &= -10 \\ &= 2(-1) + (-8) \\ &= -2 - 8 \\ &= -10 \end{aligned}$$

$\therefore$ L.S. = R.S. in both equations, $(-1, -8)$ is the solution.